

# Homework Complexity IBC028

To be handed in on the exercise session of February 15, 2019. Deadline: 11:00 AM.

This is the first set of homework exercises.

By handing in these homework exercises an extra bonus can be obtained for the examination: one full point if both sets of homeworks exercise are done perfectly, and otherwise a corresponding part of one point.

## Exercise 1.

Given is  $T(n) = T(n/2) + T(n/3) + \Theta(n)$ .

Prove that  $T(n) = O(n)$  (rounding effects of  $n/2$  and  $n/3$  may be ignored).

## Exercise 2.

Given is  $T(n) = 5T(n/2) + \Theta(n^2)$ .

Prove that  $T(n) = O(n^2\sqrt{n})$  and  $T(n) = \Omega(n^2 \log n)$ .

## Exercise 3.

For  $i = 2, 3, 4, 5$  the function  $T_i$  is given by  $T_i(1) = 1$  and

$$T_i(n) = iT_i(\lfloor n/2 \rfloor) + n^2$$

if  $n > 1$ . Determine functions  $f_i$  such that  $T_i(n) = \Theta(f_i(n))$  for  $i = 2, 3, 4, 5$ .

## Exercise 4.

Given is  $T(n) = 2T(n-1) + \Theta(n)$ .

Prove that  $T(n) = O(2^{n+\log n})$ .

## Exercise 5.

Let  $T(n) = T(n/3) + T(2n/3) + 5n$ . Prove that  $T(n) = O(n \log n)$ .