Homework Complexity IBC028

To be handed in on March 1, 2019, in the exercise group, no later than 11:00 AM.

This is the second set of homework exercises.

By handing in these homework exercises an extra bonus can be obtained for the examination: one full point if both sets of homework exercises are done perfectly, and otherwise a corresponding part of one point.

Exercise 1.

Assume access to an algorithm for matrix multiplication of two $n \times n$ matrices of time complexity $\Theta(n^2\sqrt{n})$. In the lecture a sketch of an approach was given for a matrix inversion algorithm of the same complexity, based on a way to describe matrix inversion by two recursive calls of matrix inversion of $n/2 \times n/2$ matrices, plus a finite number of matrix multiplications and matrix additions. Establish whether the same conclusion can be drawn for an alternative approach using k recursive calls of matrix inversion of $n/2 \times n/2$ matrices, plus a finite number of matrix multiplications and matrix additions, for $k = 3, 4, 5, 6$.

Exercise 2.

Let $L_1, L_2 \in P$. Prove that

$$\{u \in L_1 \mid u \notin L_2\} \in P.$$

Exercise 3.

Prove or disprove: if $L_1 \leq_P L_2$ and $L_2 \subseteq L_3$, then $L_1 \leq_P L_3$.

Exercise 4.

A CNF ϕ is satisfiable if an assignment exists for which every clause in ϕ is true. A CNF ϕ is called pre-satisfiable if an assignment exists for which every clause in ϕ is true, except for at most one. We want to prove that pre-satisfiability of CNFs is NP-complete.

(a) Describe what has to be proven for this.

(b) Give the proof.