1. Streams seminar Nijmegen 22 feb 2011 - Frits Dannenberg
On Toeplitz substitutions and transducer equivalence

Let $a, b \in \Sigma^\infty$ be single-sided infinite words over some alphabet Σ. Suppose that there are some transducers A, B such that $A(a) = b$ and $B(b) = a$. Then the two sequences are *equal under transducers* and share the same *degree*. This is indicated by novel notation:

$$a \diamond b$$

The *degree of* a includes the sequences that a can both transduce to *and* which can be transduced back to a as well. It is shown in Stern et al. that the degree of the Thue-Morse sequence includes all of its arithmetic subsequences. A special notion is reserved for degrees that include *every* transduction of *every* sequence in the degree, excluding any periodic sequences. A degree is called *prime* if for every sequence in the degree the output under transduction (the *transduct*) can be transduced back to the original sequence, or is a periodic sequence. Finite sequences are counted as periodic for the sake of simplicity. A prominent result of Klop et al. is the fact that the degree of Π is prime:

$$\Pi = 10100100010000100000\ldots$$

The degree of periodic sequences is also prime, as every sequence can be transduced to a periodic sequence. More often than not, the degree of periodic sequences will not be considered as prime but rather as the *zero* or *trivial* degree. The degree of the Thue-Morse sequence includes the period-doubling sequence, and is conjectured to be prime by Klop et al. In this presentation both the degree(s) and substitutions generated by the arithmetic subsequences of the period doubling sequence are investigated. One of the results gives rise to the *Toeplitz substitutions*. No conclusive argument regarding the conjecture is given, and the results are interpretable both affirmative and negative.

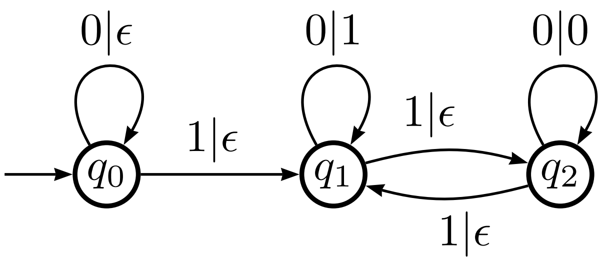

Figure 1. Example of a transducer