

Examination Automated Reasoning

Code 2IMF25, April 8, 2019, 18:00-21:00

This examination consists of 5 problems all having the same weight. The final result for this course will be the average of the result for the practical assignment and the result for this examination, as long as both results are at least 5. Here for the practical assignment the average of both parts is taken.

Problem 1.

Consider the CNF consisting of the following eight clauses

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|--------------------------------------|---------------------------------|
| (1) $p \vee \neg q \vee t$ | (5) $r \vee s \vee \neg t$ |
| (2) $\neg r \vee \neg s \vee \neg t$ | (6) $\neg p \vee q \vee t$ |
| (3) $\neg p \vee \neg q \vee t$ | (7) $p \vee q \vee t$ |
| (4) $r \vee \neg s \vee \neg t$ | (8) $\neg r \vee s \vee \neg t$ |

Determine whether this CNF is satisfiable by using the four rules UnitPropagate, Decide, Fail and Backtrack. Make clear at every step what is the corresponding list M of literals and which clause was used. (Hint: take care of choosing decision literals)

Problem 2.

Let f be a boolean function on the boolean variables p, q, r, s . If r holds then f yields true if and only if exactly one of the variables p, q, s is true. If r does not hold then f always yields true. Compute the ROBDD of f with respect to the order $p < q < r < s$.

Problem 3.

Show how by (the initialization part of) the simplex algorithm values for $x, y \geq 0$ are found satisfying

$$\begin{aligned}x + y &\geq 3 \\x + 2y &\leq 8 \\2x - y &\geq 2.\end{aligned}$$

Problem 4.

Consider the following six clauses

$$\begin{aligned}Q(A, B), \quad Q(B, C), \quad Q(C, A), \quad \neg P(A, A), \\ \neg Q(X, Y) \vee P(X, Y), \\ \neg P(X, Y) \vee \neg P(Y, Z) \vee P(X, Z),\end{aligned}$$

in which X, Y, Z denote variables and A, B, C, P, Q denote constants and relation symbols.

Prove by resolution that the conjunction of these six clauses is unsatisfiable.

Problem 5.

Given the term rewriting system R consisting of the three rules

$$x+0 \rightarrow x, \quad 0+x \rightarrow x, \quad s(x)+y \rightarrow s(y+x),$$

where $+$ is a binary symbol in infix notation, s is a unary symbol and 0 is a constant.

- Prove that R is terminating.
- Compute all critical pairs of R .
- Determine whether R is confluent.