

# Examination Automated Reasoning

Code 2IMF25, January 31, 2020, 18:00-21:00

This examination consists of 5 problems all having the same weight. The final result for this course will be the average of the result for the practical assignment and the result for this examination, as long as both results are at least 5. Here for the practical assignment the average of both parts is taken.

## Problem 1.

- a. Consider the CNF consisting of the following eight clauses

$$\begin{array}{ll} (1) & \neg p \vee t \\ (2) & p \vee q \\ (3) & \neg q \vee r \\ (4) & \neg r \vee \neg s \\ (5) & p \vee \neg r \\ (6) & r \vee \neg s \vee \neg t \\ (7) & \neg q \vee s \vee \neg t \\ (8) & \neg p \vee q \vee s \end{array}$$

Prove that this CNF is unsatisfiable by using the four rules UnitPropagate, Decide, Fail and Backtrack. As the first decision literal choose  $p^d$ ; indicate for every step the number of the clause that is used.

- b. The Tseitin transformation of the formula

$$(r \rightarrow t) \leftrightarrow ((p \wedge \neg q) \vee (r \rightarrow (s \wedge t)))$$

is a CNF. Establish the number of clauses that it has; motivate your answer.

## Problem 2.

Compute the ROBDD of

$$(s \leftrightarrow q) \wedge (r \vee \neg p)$$

with respect to the order  $p < q < r < s$ .

## Problem 3.

Apply the simplex algorithm to find the minimal value of  $2y - x$  for  $x, y \geq 0$  satisfying

$$2x + y \leq 7 \wedge 2x - y \geq 0 \wedge x - y \leq 5.$$

## Problem 4.

- a. Give a transition system with two states and a formula  $\Phi$  for which the formula

$$EG\Phi \wedge \neg AG\Phi$$

holds, motivate your answer.

- b. Describe all single resolution steps that are possible starting from the two clauses

$$P(f(x, x), x) \vee Q(x) \quad \text{and}$$

$$\neg P(x, f(y, y)) \vee \neg Q(y),$$

in which  $x, y$  are variables.

## Problem 5.

Given the term rewriting system  $R$  consisting of the two rules

$$\begin{array}{l} f(g(x, y)) \rightarrow g(h(y), x) \\ g(f(x), x) \rightarrow h(f(x)), \end{array}$$

where  $f$  and  $h$  are unary symbols and  $g$  is a binary symbol.

- a. Is  $R$  terminating? Prove your answer.
- b. Compute all non-trivial critical pairs of  $R$ .
- c. Is  $R$  confluent? Prove your answer.