Examination Automated Reasoning

Code 2IMF25, January 29, 2021, 18.00 - 21.00

This examination consists of 5 problems each having the same weight. It is a closed-book-examination: no extra material may be used. The final result for this course will be the average of the result for the practical assignment and the result for this examination, as long as both results are at least 5.

Problem 1.

Consider the CNF consisting of the following eight clauses

(1)	$\neg q \vee \neg t$	(5)	$\neg p \vee t$
(2)	$\neg p \vee \neg r$	(6)	$\neg p \vee s \vee r$
(3)	$r \vee \neg s$	(7)	$p \vee \neg r \vee \neg s$
(4)	$\neg q \vee \neg s \vee t$	(8)	$\neg p \vee q \vee s$

Determine whether this CNF is satisfiable by using the four rules UnitPropagate, Decide, Fail and Backtrack; as the first decision literal choose p^d. Make clear at every step what is the corresponding list M of literals and which clause was used.

Problem 2.

Apply the simplex algorithm to find values for $x, y, z \geq 0$ satisfying

$$\begin{array}{rcl} z & \leq & 2y + 2 \\ 4 + y & \geq & 2x \\ z + 3 & \geq & x + y. \end{array}$$

for which $2y - x$ is minimal.

Problem 3.

On the boolean variables p, q, r, s, t fix the order $p < q < r < s < t$. Compute the corresponding ROBDD of the boolean function f defined by

$$f(p, q, r, s, t) \Longleftrightarrow 2p - 2q + r - s + t > 1$$

in which false is identified with 0 and true with 1.

Problem 4.

a. Give a transition system and a formula ϕ such that $AF\phi$ holds and $EG\phi$ does not hold.

b. Transform

$$\exists y((\forall x P(x)) \rightarrow (\forall x(Q(x) \wedge P(y))))$$

to CNF by prenex normal form and skolemization.

Problem 5.

The term rewriting system R is defined to consist of the rules

$$\begin{array}{rcl} f(g(x)) & \rightarrow & g(f(x)), \\ h(f(x)) & \rightarrow & g(x), \\ g(h(x)) & \rightarrow & h(h(x)). \end{array}$$

a. Prove that R is terminating by monotone interpretations.

b. Give all non-trivial critical pairs of R.

c. Determine whether R is confluent.