Examination Complexiteit IBC028

June 25, 2019, 12.30 - 14.30

This examination consists of five problems counted by the indicated weights. The examination is 'closed book', so no use of book or notes is allowed.

For all questions: motivate your answer.

Problem 1.

(15 %) The function T is given by $T(1) = T(2) = 1$ and

$$T(n) = 3T(\lfloor n/2 \rfloor) + 2T(\lfloor n/3 \rfloor) + 4n^2$$

if $n > 2$.
Prove that $T(n) = O(n^2)$.

Problem 2.

(15 %) For $i = 1, 2, 3, 4$ the function T_i is given by $T_i(1) = 1$ and

$$T_i(n) = 3iT_i(\lfloor n/2 \rfloor) + \lfloor n^3 \log^2 n \rfloor$$

if $n > 1$.
Determine functions f_i such that $T_i(n) = \Theta(f_i(n))$ for $i = 1, 2, 3, 4$.

Problem 3.

(10 %) Give a sketch of an algorithm of complexity $O(n)$ to find an element a in an unordered sequence of n distinct numbers, such that exactly k elements of the sequence are $\leq a$.
In particular, give the corresponding recurrence relation.

Problem 4.

a. (10 %) Give the definition of the decision problem CLIQUE.

b. (15 %) Describe a transformation f from an arbitrary $\leq$3-CNF Φ (in which every clause has $\leq$ 3 literals) to a 3-CNF (in which every clause has exactly 3 distinct literals) such that Φ is satisfiable if and only if $f(\Phi)$ is satisfiable.

c. (10 %) Give an example of a quantified boolean formula with both types of quantifications that yields false.

Problem 5.

The decision problem XXX reads: given a set of inequalities of the shape

$$ax + by + cz < k$$

for $a, b, c \in \{-1, 1\}$, $k \in \mathbb{Z}$, and x, y, z are distinct variables from a set X, can we give integer values to the variables in X such that all these inequalities hold?

a. (10 %) Describe what has to be proved to conclude that XXX is NP-complete, based on the fact that 3-CNF-SAT is NP-complete.

b. (15 %) Give the proof.