Examination Complexiteit IBC028

April 5, 2018, 15.00 - 18.00

This examination consists of five problems counted by the indicated weights. The examination is 'closed book', so no use of book or notes is allowed.

For all questions: motivate your answer.

Problem 1.

(15 %) The function T is given by $T(1) = 5$ and

$$T(n) = 2T(\lfloor n/2 \rfloor) + T(\lfloor 2n/3 \rfloor) + 3n^2$$

if $n > 1$. Prove that $T(n) = O(n^2)$.

Problem 2.

(15 %) For $i = 1, 2, 3, 4$ the function T_i is given by $T_i(1) = 1$ and

$$T_i(n) = 3iT_i(\lfloor n/2 \rfloor) + n^3 \log n$$

if $n > 1$. Determine functions f_i such that $T_i(n) = \Theta(f_i(n))$ for $i = 1, 2, 3, 4$.

Problem 3.

(10 %) Give a sketch of an algorithm of complexity $O(n)$ to find an element a in an unordered sequence of n distinct numbers, such that exactly k elements of the sequence are $\leq a$. In particular, give the corresponding recurrence relation.

Problem 4.

a. (10 %) Give the definition of the decision problem ILP (integer linear programming).

b. (15 %) Describe a transformation f from an arbitrary CNF Φ to a

$\leq$3-CNF such that Φ is satisfiable if and only if $f(\Phi)$ is satisfiable.

c. (10 %) Give an example of a quantified boolean formula with both types of quantifications that yields false.

Problem 5.

The decision problem *Clique3cover* reads: given an undirected graph (V, E), are there three subsets V_1, V_2, V_3 of V such that the union of these sets is V, and for every $i = 1, 2, 3$ the graph restricted to V_i is a clique?

a. (10 %) Describe what has to be proved to conclude that Clique3cover is NP-complete, based on the fact that 3-Color is NP-complete.

b. (15 %) Give the proof.