Examination Complexiteit IBC028

July 11, 2018, 12.30 - 15.30

This examination consists of six problems counted by the indicated weights. The examination is 'closed book', so no use of book or notes is allowed.

For all questions: motivate your answer.

Problem 1.

(15 %) The function T is given by $T(1) = T(2) = 10$ and

$$T(n) = T(n-1) + T(n-2) - 1$$

if $n > 2$. Prove that $T(n) = \Omega((\frac{3}{2})^n)$.

Problem 2.

(15 %) For $i = 2, 3, 4, 5$ the function T_i is given by $T_i(1) = 1$ and

$$T_i(n) = iT_i(\lfloor n/2 \rfloor) + n^2$$

if $n > 1$. Determine functions f_i such that $T_i(n) = \Theta(f_i(n))$ for $i = 2, 3, 4, 5$.

Problem 3.

(15 %) Apply the recursion tree method on the recurrence relation

$$T(n) = T(n/2) + T(n/3) + n,$$

and give the resulting complexity.

Problem 4.

(10 %) Give a sketch of an algorithm of complexity $O(n^{2.81})$ to compute the product of two $n \times n$ matrices. In particular, give the corresponding recurrence relation and explain why this yields the resulting complexity.

Problem 5.

a. (10 %) Give the definition of NP.

b. (10 %) Give the definition of the decision problem CLIQUE.

Problem 6.

The decision problem 4-SAT reads: given a CNF in which every clause consists of exactly 4 literals, is it satisfiable?

a. (10 %) Describe what has to be proved to conclude that 4-SAT is NP-complete, based on the fact that 3-SAT is NP-complete.

b. (15 %) Give the proof.