

Examination Complexiteit IBC028

March 28, 2019, 12.30 - 14.30

This examination consists of five problems counted by the indicated weights. The examination is 'closed book', so no use of book or notes is allowed.

For all questions: motivate your answer.

Problem 1.

(15 %) The function T is defined by $T(n) = n$ if $n \leq 3$, and

$$T(n) = 2T(\lfloor n/4 \rfloor) + T(\lfloor n/2 \rfloor) + 5n$$

if $n > 3$. Prove that $T(n) = O(n \log n)$.

Problem 2.

(10 %) The function T is defined by $T(1) = 1$, and

$$T(n) = 4T(\lfloor n/2 \rfloor) + \lfloor n \log n \rfloor^2$$

if $n > 1$. Establish whether the Master theorem can be applied to find a function F such that $T(n) = \Theta(F(n))$; if so, give F , if not, explain why not.

Problem 3.

(15 %) Describe an algorithm to find in $O(n \log n)$ steps the smallest distance between two points from a given set of n points in the plane.

Problem 4.

- (10 %) Give the definition of an AVL tree.
- (15 %) Prove that \leq_P is transitive.
- (10 %) Give the definition of the decision problem *Vertex-Cover*.

Problem 5.

The decision problem *Matrix* reads: given a $k \times n$ -matrix of integers, is it possible to multiply zero or more rows by -1 such that in the resulting matrix every column contains a strictly positive number?

- (10 %) Describe what has to be proved to conclude that *Matrix* is NP-complete, based on the fact that CNF-SAT is NP-complete.
- (15 %) Give the proof. (Hint: identify rows with variables and columns with clauses)